Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

REVIEWS AND ABSTRACTS OF LITERATURE

Hauptwerke der Philosophie in original getreuen Neudrucken. Band IV. B. BOLZANO'S Wissenschaftslehre. Neu herausgegeben von Alois Höfler. Leipzig: Verlag von Felix Meiner, 1914. Erster Band. Pp. xvi + 571.

Bernhard Bolzano's scientific activities represented two important lines of thought, yet he obtained recognition in one only in comparatively recent years and long received but little attention in the other. As a precursor of Cantor in his "Paradoxieen des Unendlichen" and as a pioneer in doctrines more fully developed by Weierstrass, Bolzano has now attained deserved renown.[1] Not so well known, but it would appear, equally meritorious are Bolzano's investigations in logic represented by his *Wissenschaftslehre.* Latterly, this work, too, has been rescued from obscurity and made the basis of study by investigators, notably Pelágyi and Bergmann.[2] The increasing importance which the Wissenschaftslehre is winning for itself, combined with the scarcity of the original treatise, has made a new edition most desirable. It is gratifying, therefore, that a reprint has been undertaken.

The Wissenschaftslehre bears that formal and systematic character which is traditional among European scholars. The first volume consists of an *introduction,* a *fundamental theory,* and the first part of an *elementary theory.* The introduction is concerned with the meaning of a scientific theory and the relation of logic to science. The fundamental theory develops a doctrine of the existence and knowledge of truth. The first part of the elementary theory contains a discussion of representations.

Science, according to Bolzano, is a mere complex of truths irrespective of the order of the latter. These truths do not imply a reference to their knowledge and assumption by individuals; they are not subject to change; in fact, they are non-existential in the sense that they involve no reference to space or time. Science is here taken in an objective sense, or science *in itself.* The Wissenschaftslehre is that science usually termed logic; the task of logic is to provide rules for the analysis of truth as a whole into parts and for the exposition of each part, that is, logic is essentially the theory of scientific exposition. Bolzano suggests that logic might be called *formal* in that it deals only with expressions of the type "Some A are B," which he calls "forms" of propositions. These forms are genera of propositions and are not themselves individual, completely determined propositions; they have undetermined components such as the "A" and "B" in "Some A are B." In a subsequent note (p. 393) Bolzano specifically says that "form" is a mere (p. 247) conjunction of words or signs by means of which a certain kind of representations, propositions, and deductions may be exhibited. The general form of every[3] proposition (p. 380), for example, is "A possesses B." It is clear from Bolzano's description of species and genus (§ 106) and an incidental remark concerning a complex of true propositions (p. 111) that the "form" of propositions is not a proposition, but the designation of a representation with propositions as objects. Bolzano's "form" of propositions seems to parallel[4] Russell's *propositional function.* After considering the formal character of logic Bolzano discusses its independence from other sciences. It is maintained (p. 53) that there are few, if any, independent sciences; in particular, logic is dependent on psychology (pp. 14, 54, 227).

---

[1] *Cf.* J. T. Merz, "History of European Thought of the Nineteenth Century," Vol. II.: "Development of Mathematical Thought," page 709.

[2] *Cf.* M. Pelágyi, "Kant und Bolzano" Halle, 1902; H. Bergmann, "Das philosophische Werk Bolzanos" Halle, 1909.

[3] Bolzano thus assumes that all propositions are reducible to the subject-predicate form. On this point see Russell, "Principles of Mathematics" page 28, § 33.

In the fundamental theory Bolzano distinguishes between the proposition as such, as verbal, *i. e.,* its expression by individuals and as mental, *i. e.,* its mere representation in thought. For example, "God is omnipresent" is a true verbal proposition, "a square is round" is a false proposition; but "the round square," "the omnipresent God" are not propositions, but representations (presumably of propositions; *cf.* "that which is round and square") and are neither false nor true. Bolzano remarks (p. 79) that the distinction between a proposition as such and its representation, between a thing and its concept, is necessary to avoid errors. As an illustration he cites "What I now say is false" as a false proposition whose subject is not a proposition, but a representation of a proposition. The latter proposition, he observes, is one of a class of propositions whose subject or predicate involves a reference to the propositions themselves or some one of their components (*cf.* pp. 75, 222, 265) so that a modification of their predicate induces a modification of their subject. Such propositions may have the further property that their contradictory opposite is not obtainable by prefixing "not" to their predicate; to give another illustration, "the number of words in this sentence is ten" and "the number of words in this sentence is not ten" are both false propositions. To return to Bolzano's general exposition: propositions *in themselves* are assertions (pp. 77, 115) of truth or falsehood irrespective of verbal expression or of mental representation (p. 78). Between the constituent representations in a proposition there is a relation (p. 97). Subordinate to the concept "proposition in itself" is the concept "truth in itself": all truths in themselves are propositions in themselves. By a truth in itself is meant a proposition which "asserts something as it really is." The words in this definition have a technical signification of which Bolzano believes he takes sufficient account by remarking that for a truth in itself it is undetermined whether the proposition is thought or verbally expressed (p. 115). Truths in themselves are, for example, the truths of religion, ethics, mathematics, and metaphysics (p. 113). Such truths must be carefully distinguished from known truths. Bolzano bases this distinction on the principle that two concepts may be convertible and yet distinct. Thus all truths in themselves are known by God, since God is omniscient, and are, therefore, known truths: but the concept of a known truth is distinct from the concept of a truth in itself.

---

[4] Russell indeed states (*loc. cit.,* page 13) explicitly: "A propositional function is, as it were, a schematic form standing for any one of a whole class of propositions." The remarks of Bergmann (*loc. cit.,* page 87) will be found suggestive in this connection, but should be read with caution.

The distinction in question is analogous to the difference between two equivalent definitions of a line in geometry or presumably the difference between cardinal and ordinal numbers in arithmetic, etc. Historically, the concepts " proposition in itself " and " truth in itself " may be traced to Leibniz, among others (pp. 85, 120).

In the first part of the elementary theory Bolzano defines " representations in themselves " (*cf.* § 56, p. 244) and gives an elaborate classification of these concepts. " Proposition in itself " and " representation in itself " are mutually exclusive; however, part of a representation may be a proposition, and conversely representations have the (characteristic) property that they are, or may be, parts of propositions (pp. 221, 238). In particular, then, anything that may be made the subject of a proposition is a representation. Any word designates a representation in itself, and if the former is unambiguous, the latter is unique (pp. 217, 218). Bolzano's classification of representations is made first with regard to those internal qualities (*Beschaffenheiten*) or *properties* which are possessed by representations without reference to other objects and second with respect to those external qualities or *relations* of which the contrary is valid. The first basis of classification leads Bolzano to discuss simple and complex, concrete and abstract, real and imaginary representations. Representations in themselves may or may not refer to an object. The object of a representation is distinct from the latter. In illustration, Bolzano states: the representations " nothing " and " round square " have no objects; the representation " this A " in an immediate sense (§ 59) has but one object; " integer lying between one and ten " has a finite number of objects and the representation " line " has an infinite number of objects. Simple representations having but one object are called *perceptions* (pp. 327, 328); representations which are not perceptions, wholly or in part, are called *conceptions* (p. 330). In § 79 a digression is made dealing with the perceptual and conceptual aspects of space and time (*cf.* p. 370). In general, representations having at least one object have a *domain*, which is described (pp. 297, 298) as that property of a representation possessing objects in virtue of which it represents these and no others. The domain of each representation is unique; a representation with no object (§ 67) may be said to have an empty domain. If the domain of a complex representation remains invariant when parts of the representation are removed, the representation is said to be redundant (§ 69). In close connection with the redundancy of a representation is Bolzano's (functional) conception of the variability of a representation as dependent on an independent variable, that is, the derivation of new representations from a given representation by replacing some constituent of the given representation with *arbitrary* representations (p. 314). This quasi-functional character of a representation allows Bolzano to extend (p. 311) the conception of redundancy to representations with no object, *e. g.*, imaginary representations. Thus " a round and four cornered square " might be considered redundant with reference to the constituent " four cornered " since the representation in question yields properly redundant representations by the suitable substitutions of representations for " round."

An important property of the domain of a representation is its *extension* (§§ 66, 93) which is a magnitude (*cf.* § 87). If a representation has a finite number of objects the extension of its domain may be determined absolutely by the individual specification (enumeration) of these objects. In the case of an infinite number of objects the extension can be determined only incompletely[5] and relatively, that is, by comparison with the extension of other infinite domains: by specifying that the domain of the representation A has the same extension as the domain of the representation B, or the domain of A is part of the domain of B, etc. (p. 299; *cf.* §§ 87, 93). Here Bolzano is in touch with his "Paradoxieen des Unendlichen." In fact the extension of a domain through its underlying principle stated by Bolzano (pp. 302, 303) anticipates[6] a standard conception of the modern theory of assemblages. The domain of a representation in its purely logical aspect, moreover, is itself of the highest importance and has been dwelt upon by recent writers, though in a manner distinctly inferior to Bolzano's exposition.

Before proceeding to the second basis of classification of representations, Bolzano describes representations of property and relation (§ 80). If a whole consisting of parts has a quality which is distinct from the quality of each part, then the quality possessed by the whole is a *relation* between the parts. The relation is, further, a function, as it were, of related variables, the latter being restricted to objects of the same kind (p. 381). Clearly, the preceding conception of relation is opposed to the view of the neo-realists such as G. E. Moore and others. However, Bolzano calls relations external qualities; and non-external or internal qualities are called properties. He distinguishes between symmetrical and unsymmetrical relations and discusses the possibility of a relation between an object and itself (p. 384). In the concluding two chapters, Bolzano compares representations first, with reference to relations between themselves, and second, with reference to their relations to other objects. Perhaps the most interesting relations discussed are those of extension (§ 93), convertibility (§ 96), inconsistency[7] (§ 103, *cf.* pp. 480, 509), and opposition (§ 107). Analogous to the generalization of the concept redundancy (§ 69) Bolzano generalizes convertibility and other relations so as to apply to representations having no objects (§ 108). In § 109 (*cf.* § 55) a kind of correspondence between true and false propositions, and "correct" and "incorrect" representations is established.

These are the prominent features of the first volume of the Wissenschaftslehre. The work abounds with valuable historical references and throughout his discussion Bolzano has endeavored to illustrate his arguments with examples from elementary mathematics. Apparently Bolzano was guided to a considerable degree by mathematical conceptions, attempting to bring into logic the precision and standardization[8] of deductive mathematics. Some of Bolzano's illustrations, however, emphasize the incompleteness of mathematical science in his time; for example, his interpretation of $\sqrt{-1}$ as an imaginary representation, on a par with a round square, presents a problem which has since been solved by the general theory of complex numbers.

---

[5] From the above remarks of Bolzano it would seem that Russell is wrong when he says (''Principles of Mathematics,'' page 70) that Bolzano did not recognize the impracticability of applying enumeration to infinite systems.

[6] *Cf.* the Borel-Lebesgue ''measure'' of an assemblage; Hobson, ''Theory of Functions of a Real Variable,'' § 81.

[7] *Cf.* Royce, *Transactions of the American Mathematical Society*, 1905, pages 359, 365, etc.

It has been said that Bolzano's Wissenschaftslehre prepared the way for a "phenomenological" method in logic and the theory of knowledge. From this point of view, a comparison of the Wissenschaftslehre with Husserl's "Logische Untersuchungen"[9] and Meinong's "Gegenstandstheorie"[10] facilitates an insight into these authors. It is instructive also to study Bolzano's work in its points of contact with Russell's "Principles of Mathematics." Although Russell has commented on the excellence of the "Paradoxieen des Unendlichen" it is strange that he does not seem explicitly to mention the Wissenschaftslehre in his book. Much of the ingenuity shown by Russell in discussing such problems as *meaning* and *denoting, propositional concept* (logical subject, verbal noun, and *proposition, whole* and *part, conceptual* and *real analysis* will be found to be characteristic of Bolzano. Striking differences between the two authors are contained in their views of the relevance of psychology to logic, the nature of relation, and the theory of subject and predicate. Russell,[11] has compared his doctrines with those of Frege. We are enabled to infer resemblances between Frege and Bolzano by means of the interesting controversy the former author has had with Korselt in the "Jahresbericht der Deutschen Mathematiker Vereinigung."[12]

A few words concerning Bolzano's historical importance may be added. It is doubtful whether Bolzano is the successor of Leibniz, as has been suggested. Nevertheless, he deserves to be associated with Leibniz and also Descartes, inasmuch as those great thinkers devoted themselves both to philosophy and mathematics without unfavorable discrimination and thus occupied a position which seems certain of a vigorous revival and promises to be of extraordinary fecundity for both disciplines.

ARTHUR R. SCHWEITZER.

CHICAGO.

What Ought I to Do? An Inquiry into the Nature and Kinds of Virtue and into the Sanctions, Aims, and Values of the Moral Life. GEORGE TRUMBULL LADD. New York: Longmans, Green and Company. 1915. Pp. 311.

This work is the second in a series of four volumes addressed rather to the larger educated public than to students of philosophy in the more restricted and technical sense. The first volume dealt with the problem

---

[8] See, for instance, Bolzano, *loc. cit.*, page 509.

[9] *Cf.* Vol. II., page 7; Vol. I., page 227.

[10] *Cf.* the review by Professor Perry, this JOURNAL, Vol. XII., pages 301–305.

[11] *Loc. cit.*, pages vi, 501.

[12] Vol. 12 (1903), 15 (1906), 17 (1908).